Shannon weights for binary sources of zero entropy

Instances of dynamical sources

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Joint work with

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General aim.

Shannon weights are an extension of Shannon entropy

- defined even when Shannon entropy h is zero
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Plan of the talk

(I) Main facts on sources and precise definition of Shannon weights

(II) A main tool in our work : Generating functions

(III) Sources of interest : dynamical sources with indifferent fixed points

(IV) Exhibiting sources with prescribed Shannon weights

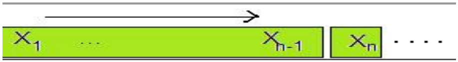
(I) Main facts on sources – Shannon weights

Source on an alphabet Σ

This is a probabilistic mechanism

which emits at each time $n \geq 1$ a symbol $\alpha_n \in \Sigma$

possibly correlated with the previous emitted symbols $\alpha_1, \ldots, \alpha_{n-1}$.



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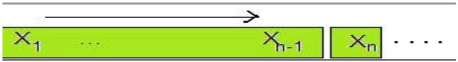

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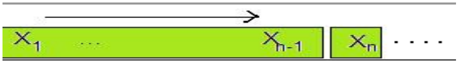

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(2) The space $\Sigma^{\mathbb{N}}$ gives rise to a family of spaces $(\Sigma^k)_{k \geq 0}$.

Each Σ^k deals with the prefixes w of length k,

their cylinders $\langle w \rangle = \{\alpha \in \Sigma^{\mathbb{N}} \mid \alpha$ begins with the finite prefix $w \in \Sigma^\star]$

and their probabilities $\mu\langle w \rangle$

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For instance we wish to build a source with

$$\underline{m}_\mu(k) = \Theta(\sqrt{k}), \quad \text{or} \quad \underline{m}_\mu(k) = \Theta(k/\log^3 k) \quad \text{or} \ldots$$

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$$w = (0^{u_1 - 1}1) \dots (0^{u_{\ell-1} - 1}1)\ (0^{u_\ell - 1}1)\ 0^{u_0}$$

... that decomposes into blocks $\mathcal{U}_n = 0^{n-1}1$ with $n \geq 1$

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- the number $n(w)$ of ones in the prefix w,

and its average on the prefixes of length k

$$\underline{n}_\mu(k) = \sum_{w \in \Sigma^k} \mu\langle w\rangle\, n(w)$$

(II) Generating functions

What can be expected on the three sequences $[q_\mu(k), \underline{n}_\mu(k), \underline{m}_\mu(k)]$?

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If the block source $\mathcal{B}$ is of finite entropy $\mathcal{E}(\mathcal{B})$,

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Our main tools : three generating functions, one for each "cost",

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We obtain relations between these gf's ....

from which we deduce relations between their coefficients

Our first result.

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Relating the behaviour of generating functions
when the real v tends vers 1^-

Result 1. There exist (dynamical) sources with zero entropy for which

- the block source $\mathcal{B}$ is a "good" source, with entropy $\mathcal{E}(\mathcal{B})$,
- results of renewal flavour hold.

There is a constant A_μ for which, when the real v tends to 1^-

$$(1-v)N_\mu(v) \sim_{1^-} A_\mu \frac{1}{(1-v)Q_\mu(v)} \qquad (1-v)M_\mu(v) \sim_{1^-} A_\mu \frac{\mathcal{E}(\mathcal{B})}{(1-v)Q_\mu(v)},$$

and thus $$(1-v)M_\mu(v) \sim_{1^-} \mathcal{E}(\mathcal{B})(1-v)N_\mu(v)$$

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The first estimate is of renewal type.

The third estimate is obtained in an indirect way

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– a sequence $a(n) \geq 0$, its partial sums $A_n = a(0) + a(1) + \ldots + a(n-1)$, the generating function $A(v) = \sum_n a(n)\, v^n$, assumed to converge for $v \in [0, 1[$

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Then the assertions (1) and (2) are equivalent:

$$(1) \quad A(v) \sim_{v \to 1^-} \frac{1}{(1-v)^\rho} U\left(\frac{1}{1-v}\right) \quad \Longleftrightarrow \quad (2) \quad A_n \sim_{n \to \infty} \frac{n^\rho}{\Gamma(\rho+1)} U(n)$$

Abelian Theorem : $(2) \Longrightarrow (1)$ Tauberian Theorem : $(1) \Longrightarrow (2)$

Strategy with a "round trip". Our final result

We assume a source for which Result 1 holds

together with Result 2 : " there exists asymptotics for the sequence $q_\mu(n)$"

With a round trip strategy based on the AT Theorem, we obtain the final Result 3 that describes the asymptotic behaviour of Shannon weights.

Result 3.

Result 2 AT Theorem

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Result 3		

We are then led to study sources for which both Result 1 and Result 2 hold

(III) Dynamical sources with indifferent fixed point

A dynamical source $\mathcal{S}$ associated with a binary dynamical system $(\mathcal{I}, \mathcal{T})$.

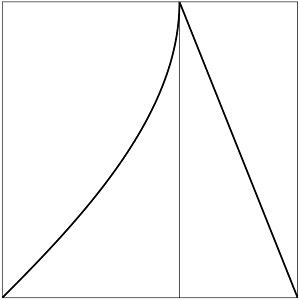
- A partition of $\mathcal{I} := [0,1]$ with $\mathcal{I}_0 = [0,c]$ and $\mathcal{I}_1 =]c,1]$
- A shift T has two bijective branches of class $\mathcal{C}^2$

$$A : \mathcal{I}_0 \to \mathcal{I} \text{ and } B : \mathcal{I}_1 \to \mathcal{I}$$

with their inverse denoted as a, b

- An encoding $\sigma : \mathcal{I} \to \{0,1\}$ with $\sigma(x) = j$ iff $x \in \mathcal{I}_j$
- Tent shape.

With a measure μ on $\mathcal{I}$, this defines the source $\mathcal{S}$
which emits words $M(x) = (\sigma(x), \sigma(Tx), \ldots \sigma(T^k x), \ldots)$



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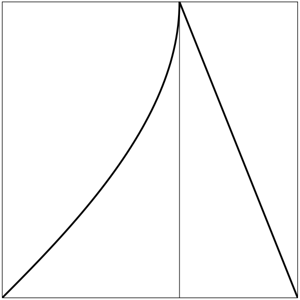

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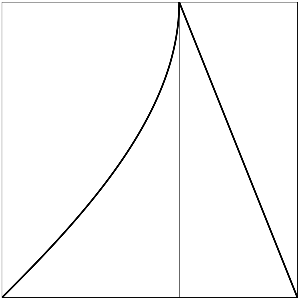

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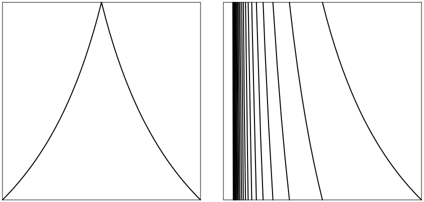

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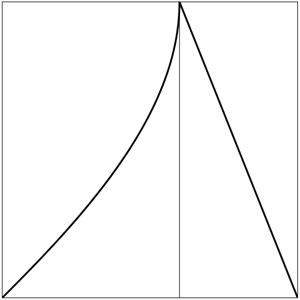

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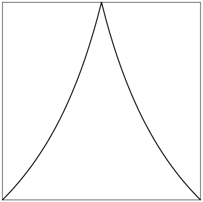
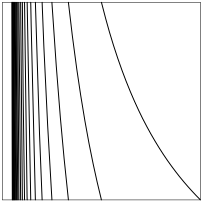
A particular instance....



Farey source



Block source of Farey = Gauss

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- What is the behaviour of $(I - \mathbb{G}_{v,1})^{-1}$ when $v \to 1^-$?

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This proves the estimates stated in our Result 1 !

A good class of dynamical sources with indifferent fixed points

These sources occur

– when the inverse branches a and b are strictly contracting on $]0, 1]$,

$$|a'(x)| < 1 \text{ and } |b'(x)| < 1 \text{ except perhaps at } x = 0$$

– the branch a has a fixed indifferent fixed point at 0: $a(0) = 0, a'(0) = 1$.

We deal in particular with the subclass $\mathcal{DRIL}(\gamma, \delta)$ $(\gamma \geq 1)$ which gathers the systems for which the branch a satisfies $\quad a'(x) = 1 - x^\gamma V_\delta(x),$

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Two important results for the subclass $\mathcal{DRIL}(\gamma, \delta)$.

(a) Result 2 [Aaronson 1980].

There is an asymptotics behaviour for the sequence $q(n)$

(b) Result 1. [Ours, this work]

The operator $\mathbb{G}_{v,s}$ of the source $\mathcal{B}$ admits dominant spectral properties

(IV) Conclusion: Exhibiting sources with prescribed Shannon weights

We deal with sources of the Class $\mathcal{DRIL}(\gamma, \delta)$ and apply Results 1, 2 and thus Result 3

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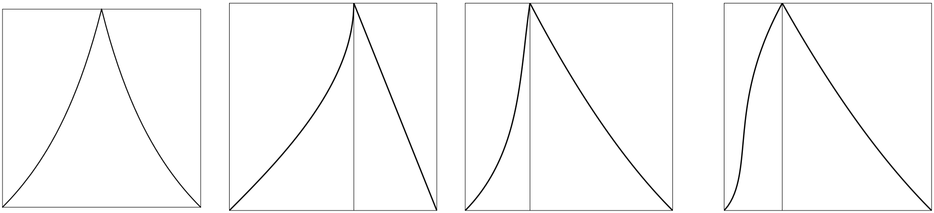

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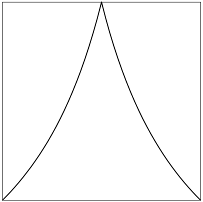

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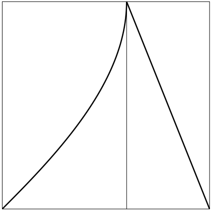

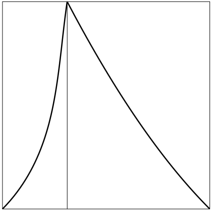

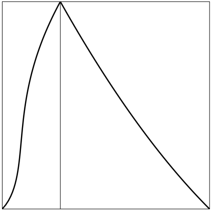

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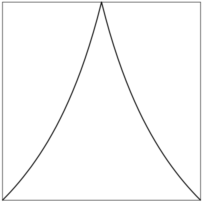

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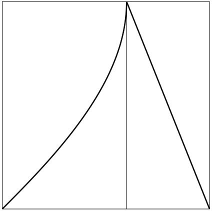
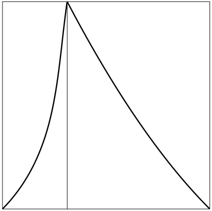
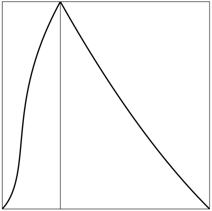


$$n^{1/2}$$

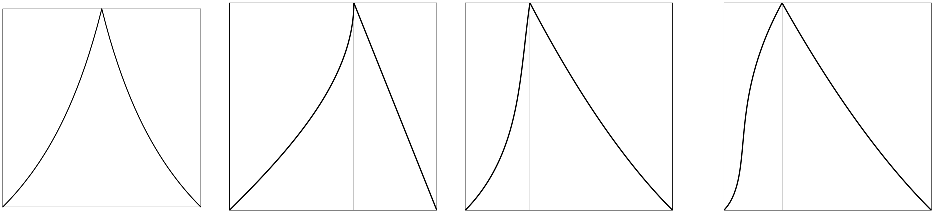




(IV) Conclusion: Exhibiting sources with prescribed Shannon weights

We deal with sources of the Class $\mathcal{DRIL}(\gamma, \delta)$ and apply Results 1, 2 and thus Result 3

The class $\mathcal{DRIL}(\gamma, \delta)$ provides instances of binary dynamical sources with Shannon weights of various orders, depending on (γ, δ)



$\frac{n}{\log n}$ $\qquad n^{1/2}$ $\qquad n^{10/11}(\log n)^{10/11}$

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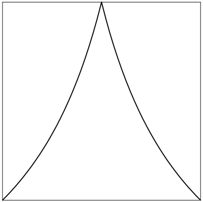

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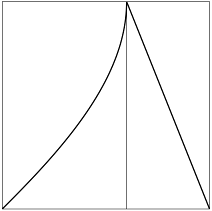

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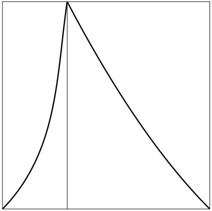
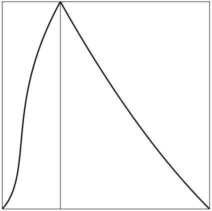


$$n^{10/11}(\log n)^{10/11}$$



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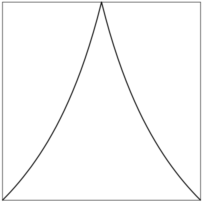

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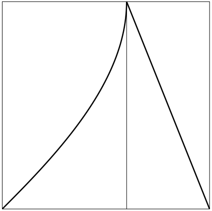
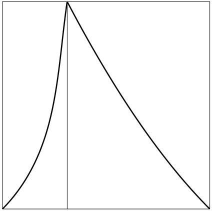
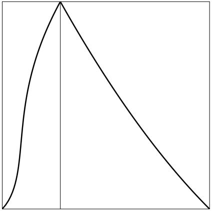


$n^{1/2}$



$n^{10/11}(\log n)^{10/11}$



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Merci beaucoup pour votre attention !